

Lepton masses and mixings in orbifold models with three Higgs families

N. Escudero,^{ab} C. Muñoz^{ab} and A.M. Teixeira^c

^a*Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, Cantoblanco, E-28049 Madrid, Spain*

^b*Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, E-28049 Madrid, Spain*

^c*Laboratoire de Physique Théorique, UMR 8627, Université de Paris-Sud XI, Bâtiment 201, F-91405 Orsay Cedex, France*

E-mail: nicolas.escudero@uam.es, carlos.munoz@uam.es, Ana.Teixeira@th.u-psud.fr

ABSTRACT: We analyse the phenomenological viability of heterotic Z_3 orbifolds with two Wilson lines, which naturally predict three supersymmetric families of matter and Higgs fields. Given that these models can accommodate realistic scenarios for the quark sector avoiding potentially dangerous flavour-changing neutral currents, we now address the leptonic sector, finding that viable orbifold configurations can in principle be obtained. In particular, it is possible to accommodate present data on charged lepton masses, while avoiding conflict with lepton flavour-violating decays. Concerning the generation of neutrino masses and mixings, we find that Z_3 orbifolds offer several interesting possibilities.

KEYWORDS: Compactification and String Models, Neutrino Physics, Higgs Physics, Quark Masses and SM Parameters.

Contents

1. Introduction	1
2. Yukawa couplings in Z_3 orbifold models	3
3. The extended Higgs sector	6
4. Charged leptons	8
4.1 Charged lepton masses	8
4.2 Tree-level lepton flavour violation	11
5. Orbifold analysis at the string scale	14
6. Neutrinos	17
6.1 Dirac neutrino masses without seesaw	17
6.2 Neutrino masses via a type-I seesaw	19
6.3 A viable seesaw from the FI breaking	23
7. Conclusions	24

1. Introduction

At present, string theory is the only candidate to unify all known interactions (strong, electroweak and gravitational) in a consistent way. In addition to necessarily containing the standard model (SM) as its low-energy limit, string theory is expected to address some of its puzzles, namely the nature of masses, mixings and number of families.

From observation, we have firm evidence that Nature contains three families of quarks and leptons, with peculiar mass hierarchies. Moreover, the flavour structure in both quark and lepton sectors is far from trivial, as exhibited by the current bounds on the quark [1] and lepton [2–5] mixing matrices. At present, we are still lacking two ingredients that are instrumental in understanding flavour dynamics: the discovery of the Higgs boson (which would thus confirm the mechanism of mass generation), and the precise knowledge of how fermions and Higgs scalars interact, that is, the Yukawa couplings of the fundamental theory. In this sense, a theory that aims at successfully explaining the observed fermion spectra, must necessarily be predictive regarding the Yukawa couplings.

Within the context of string theory, a natural and aesthetic solution may arise from Z_3 orbifold compactifications. In fact, a very interesting way to obtain a four dimensional effective theory is the compactification of the $E_8 \times E_8$ heterotic string [6] on six-dimensional orbifolds [7], and this has proved to be a very successful attempt at finding the superstring

standard model [8–35]. As it was shown in [9, 11], the use of two Wilson lines [7, 8] on the torus defining a symmetric Z_3 orbifold can give rise to supersymmetric (SUSY) models with $SU(3) \times SU(2) \times U(1)^n$ gauge group and three families of chiral particles with the correct $SU(3) \times SU(2)$ quantum numbers. These models present very attractive features from a phenomenological point of view. One of the $U(1)$ s of the extended gauge group is in general anomalous, and it can induce a Fayet-Iliopoulos (FI) D -term [36–39] that would break SUSY at very high energies (FI scale $\sim \mathcal{O}(10^{16-17} \text{ GeV})$). To preserve SUSY, some fields will develop a vacuum expectation value (VEV) to cancel the undesirable D -term. The FI mechanism allows to break the gauge group down to $SU(3)_c \times SU(2)_L \times U(1)_Y$, as shown in refs. [16, 17] and [12].

Orbifold compactifications offer some remarkable properties in relation to the flavour problem. In particular, they provide a geometric mechanism to generate the mass hierarchy for quarks and leptons [40–44] through renormalisable Yukawa couplings. Z_n orbifolds have twisted fields which are attached to the orbifold fixed points. Fields at different fixed points may communicate with each other only by world sheet instantons. The resulting renormalisable Yukawa couplings can be explicitly computed [40, 41, 45–51] and receive exponential suppression factors that depend on the distance between the fixed points to which the relevant fields are attached. These distances can be varied by giving different VEVs to the T -moduli associated with the size and the shape of the orbifold.

Orbifold scenarios with purely renormalisable Yukawa couplings, and which are a priori successful from a phenomenological point of view can be obtained if one relaxes the requirement of a minimal SUSY matter content (with just two Higgs doublets) [49]. In fact, Z_3 orbifolds with two Wilson lines naturally contain three families of everything, including Higgses, and allow mechanisms for obtaining realistic fermion masses and mixings, entirely at the renormalisable level. The analysis carried out in ref. [49] was a phenomenological “bottom-up” one. That is the particles were assigned to fixed points in a way that can reproduce the experimental data. In addition the scheme relied on the mixing between fields due to the Fayet-Iliopoulos breaking. In this scenario the low-energy theory corresponds to the minimal supersymmetric standard model (MSSM) with three Higgs families.

Whether or not such scenarios are indeed viable from a phenomenological point of view is a question that deserves careful consideration. Not only should these scenarios correctly reproduce quark and lepton masses and mixings, but they should also comply with the current bounds on flavour-changing neutral currents (FCNCs). Indeed, having three Higgs families with non-trivial couplings to matter potentially gives rise to FCNCs at the tree-level [52–55]. In previous studies [56], we have addressed the phenomenology of the Higgs and quark sectors of Z_3 models with two Wilson lines. We have verified that quark masses and mixings could be reproduced, and that FCNCs in the neutral K -, B - and D -sectors could be avoided by a fairly light Higgs spectrum. It is worth recalling that, after fitting the quark data, the free parameters defining the orbifold geometry are already very constrained.

In this work, we complete our previous analyses, by investigating whether or not the Z_3 orbifolds can also succeed in accommodating present data on charged lepton masses, while avoiding conflict with lepton flavour violating (LFV) tree-level processes, such as three-

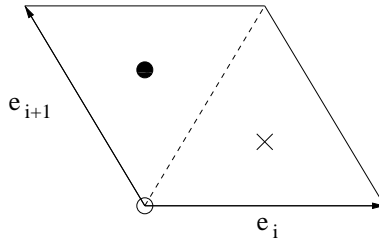


Figure 1: Two dimensional sublattice of the Z_3 orbifold, symbolically denoting the fixed points as (\circ, \bullet, \times) .

body decays. Finally, and most interesting, is the question of complying with experimental data on neutrino mass squared differences and mixing angles. This analysis is particularly challenging since, as we will discuss, orbifolds offer a variety of possibilities to account for the smallness of neutrino masses.

Let us recall that the experimental observation of neutrino oscillations has led to extend the SM in order to accommodate non-vanishing neutrino masses. In the absence of a predictive theory for the Yukawa couplings, it is only common to argue that purely Dirac neutrinos pose a naturalness problem, in the sense that the associated couplings are extremely tiny. Moreover, and contrary to what is observed in the quark sector, the leptonic mixing, parameterised by the Maki-Nagakawa-Sakata matrix, U_{MNS} [57, 58], is nearly maximal. Z_3 orbifolds offer several possibilities for the generation of neutrino masses and mixings, ranging from a purely Dirac formulation, to several implementations of a type-I seesaw mechanism [59]. Here we will argue on the viability of each possibility.

This paper is organised as follows. In section 2 we conduct a brief overview of the main properties of Yukawa couplings in this class of Z_3 orbifold compactifications. In section 3, we summarise the most relevant features of the extended Higgs sector. The additional constraints on the orbifold parameters obtained from reproducing the charged lepton masses are presented in section 4, where we also discuss the tree-level contributions to LFV processes. In section 5, we comment on the implications of the phenomenologically derived constraints regarding the properties of the compact space. Section 6 is devoted to the discussion of the viability of several mechanisms regarding the generation of neutrino masses and mixings. We summarise our findings in section 7.

2. Yukawa couplings in Z_3 orbifold models

In this section we briefly review the most relevant features of the geometrical construction of the Z_3 orbifold leading to the computation of the fermion mass matrices.

The construction is made by dividing the R^6 space by a $[\text{SU}(3)]^3$ root lattice modded by the point group (P) with generator θ , where the action of θ on the lattice basis is $\theta e_i = e_{i+1}$, $\theta e_{i+1} = -(e_i + e_{i+1})$, with $i = 1, 3, 5$. The two-dimensional sublattices associated to $[\text{SU}(3)]^3$ are presented in figure 1.

In orbifold compactifications, twisted strings appear attached to fixed points under the point group. In the case of the Z_3 orbifold there are 27 fixed points under P, and therefore 27 twisted sectors. We will denote the three fixed points of each two-dimensional lattice as (\circ, \bullet, \times) . The general form of the Yukawa couplings between the twisted fields in Z_3 orbifolds is given by the Jacobi theta function (see, for example, the appendix of ref. [48]). These Yukawa couplings contain suppression factors that depend on the relative positions of the fixed points to which the fields involved in the coupling are attached, and on the size and shape of the orbifold (i.e. the deformation parameters). Imposing invariance under the point group reduces these parameters to nine: the three radii of the sublattices and the six angles between complex planes. The latter parameters correspond to the VEVs of nine singlet fields appearing in the spectrum of the untwisted sector, and which have perturbatively flat potentials. These so-called moduli fields are usually denoted by T . In the most simple assumption, i.e. when the six angles are zero, there are only three relevant parameters which characterise the whole compact space.

Assuming that the two non-vanishing Wilson lines correspond to the first and second sublattices, then the 27 twisted sectors come in nine sets with three equivalent sectors in each one. The three generations of matter (including Higgses) correspond to changing the third sublattice component (\circ, \bullet, \times) of the fixed point, while keeping the other two fixed. Consider for example the following assignments of observable matter to fixed point components in the first two sublattices,

$$\begin{array}{lll}
 Q \leftrightarrow \circ\circ & u^c \leftrightarrow \circ\circ & d^c \leftrightarrow \times\circ \\
 L \leftrightarrow \bullet\bullet & e^c \leftrightarrow \bullet\times & \nu^c \leftrightarrow \times\times \\
 H^u \leftrightarrow \circ\circ & H^d \leftrightarrow \bullet\circ & .
 \end{array} \tag{2.1}$$

In this case, the fermion mass matrices before taking into account the effect of the FI breaking, are given by the following expression [49]:

$$\begin{aligned}
 \mathcal{M}^u &= g N A^u, & \mathcal{M}^d &= g N \varepsilon_1 A^d, \\
 \mathcal{M}^\nu &= g N \varepsilon_1 \varepsilon_3 A^u, & \mathcal{M}^e &= g N \varepsilon_3 A^d,
 \end{aligned} \tag{2.2}$$

where

$$A^u = \begin{pmatrix} w_2 & w_6 \varepsilon_5 & w_4 \varepsilon_5 \\ w_6 \varepsilon_5 & w_4 & w_2 \varepsilon_5 \\ w_4 \varepsilon_5 & w_2 \varepsilon_5 & w_6 \end{pmatrix}, \quad A^d = \begin{pmatrix} w_1 & w_5 \varepsilon_5 & w_3 \varepsilon_5 \\ w_5 \varepsilon_5 & w_3 & w_1 \varepsilon_5 \\ w_3 \varepsilon_5 & w_1 \varepsilon_5 & w_5 \end{pmatrix}, \tag{2.3}$$

g is the gauge coupling constant, and N is related to the volume of the Z_3 lattice unit cell such that $g N \approx 1$. In the above matrices w_i denote the VEVs of the neutral components of the six Higgs doublet fields. Since we are assuming an orthogonal lattice, i.e. with the six angles equal to zero, only the diagonal moduli T_i (which are related to the radii of the three sublattices) contribute to the Yukawa couplings, through the suppression factors ε_i

$$\varepsilon_i \approx 3 e^{-\frac{2\pi}{3} T_i}, \quad i, j = 1, 3, 5. \tag{2.4}$$

The existence of an anomalous $U(1)$ in the extended $SU(3) \times SU(2) \times U(1)^n$ gauge group generates a Fayet-Iliopoulos D -term which could in principle break SUSY at energies close

to the string scale. This term can be cancelled when scalar fields (C_i), which are singlets under $SU(3) \times SU(2)$, develop large VEVs (10^{16-17} GeV). The VEVs of these fields (c_i), have several important effects. Firstly, they break the original $SU(3) \times SU(2) \times U(1)^n$ gauge group down to the (MS)SM $SU(3) \times SU(2) \times U(1)$. Secondly, they induce very large effective mass terms for many particles (vector-like triplets and doublets, as well as singlets), which thus decouple from the low-energy theory. Even so, the SM-like matter remains massless, surviving as the zero mass mode of combinations with the other (massive) states. All these effects modify the mass matrices of the low-energy effective theory (see eq. (2.2)), which, for the example studied in [49], are now given by

$$\begin{aligned} \mathcal{M}^u &= g N a^{u^c} A^u B^{u^c}, & \mathcal{M}^d &= g N \varepsilon_1 a^{d^c} A^d B^{d^c}, \\ \mathcal{M}^\nu &= g N \varepsilon_1 \varepsilon_3 a^L a^{\nu^c} B^L A^u B^{\nu^c}, & \mathcal{M}^e &= g N \varepsilon_3 a^L a^{e^c} B^L A^d B^{e^c}, \end{aligned} \quad (2.5)$$

where $A^{u,d}$ are the Higgs VEV matrices prior to FI breaking (see eq. (2.3)), a^f is given by

$$a^f = \frac{\hat{c}_2^f}{\sqrt{|\hat{c}_1^f|^2 + |\hat{c}_2^f|^2}}, \quad (2.6)$$

with $f = (u^c, d^c, L, e^c, \nu^c)$, and B^f is the diagonal matrix defined as

$$B^f = \text{diag}(\beta^f \varepsilon_5, 1, \alpha^f / \varepsilon_5). \quad (2.7)$$

Finally

$$\alpha^f = \varepsilon_5 \sqrt{\frac{|\hat{c}_1^f|^2 + |\hat{c}_2^f|^2}{|\hat{c}_1^f \varepsilon_5|^2 + |\hat{c}_2^f|^2}}, \quad \beta^f = \sqrt{\frac{|\hat{c}_1^f|^2 + |\hat{c}_2^f|^2}{|\hat{c}_1^f|^2 + |\hat{c}_2^f \varepsilon_5|^2}}. \quad (2.8)$$

In the above, \hat{c}_i^f are derived from the VEVs of the heavy fields responsible for the FI breaking as

$$\hat{c}_1^f \equiv \varepsilon'^{(f)} c_1^f, \quad \hat{c}_2^f \equiv \varepsilon''^{(f)} c_2^f, \quad (2.9)$$

where in each case ε' and ε'' can take any of the following values:

$$\varepsilon', \varepsilon'' \equiv 1, \varepsilon_1, \varepsilon_3, \varepsilon_1 \varepsilon_3. \quad (2.10)$$

Let us also stress that one should not take $\alpha^f, \beta^f, \varepsilon_5$ and a^f as independent parameters. In fact, eqs. (2.6), (2.8) imply that

$$a^f = \frac{(1 - \alpha^{f2})^{1/2}}{\alpha^f} \frac{\varepsilon_5}{(1 - \varepsilon_5^2)^{1/2}} = \left(1 - \frac{1}{\beta^{f2}}\right)^{1/2} \frac{1}{(1 - \varepsilon_5^2)^{1/2}}, \quad (2.11)$$

so that for given values of ε_5 and α^f , β^f is fixed as

$$\beta^f = \frac{1}{\sqrt{1 + \varepsilon_5^2 \left(1 - \frac{1}{\alpha^{f2}}\right)}}. \quad (2.12)$$

The most striking effect of the FI breaking is that it enables the reconciliation of the Yukawa couplings predicted by this scenario with experiment. In particular, as it has been shown [56], the quark spectrum and a successful Cabibbo-Kobayashi-Maskawa (CKM) matrix can now be accommodated. Moreover, expanding the eigenvalues of the quark mass matrices up to leading order in ε_5 , one can derive the following relation for the Higgs VEVs in terms of the quark masses

$$\begin{aligned} \{w_1, w_3, w_5\} &= \frac{1}{gN \varepsilon_1 a^{d^c}} \left\{ \frac{1}{\varepsilon_5 \beta^{d^c}} \left(m_d + \varepsilon_5^5 \frac{m_b^2}{m_s} \right), m_s, \frac{m_b \varepsilon_5}{\alpha^{d^c}} \right\}, \\ \{w_2, w_4, w_6\} &= \frac{1}{gN a^{u^c}} \left\{ \frac{1}{\varepsilon_5 \beta^{u^c}} \left(m_u + \varepsilon_5^5 \frac{m_t^2}{m_c} \right), m_c, \frac{m_t \varepsilon_5}{\alpha^{u^c}} \right\}. \end{aligned} \quad (2.13)$$

3. The extended Higgs sector

As it has been previously discussed, this class of orbifold models contain naturally a replication of Higgs families. In this section we will summarise those features of the Higgs sector relevant for the present analysis (for a more complete study of generic SUSY models with three Higgs generations, see ref. [55]).

By construction, let us consider an orbifold scenario containing three generations of SU(2) Higgs doublet superfields, with hypercharge $-1/2$ and $+1/2$, respectively coupling to down- and up- type quarks.

$$\widehat{H}_{1(3,5)} = \begin{pmatrix} \widehat{h}_{1(3,5)}^0 \\ \widehat{h}_{1(3,5)}^- \end{pmatrix}, \quad \widehat{H}_{2(4,6)} = \begin{pmatrix} \widehat{h}_{2(4,6)}^+ \\ \widehat{h}_{2(4,6)}^0 \end{pmatrix}. \quad (3.1)$$

We assume the most general form of the superpotential for the quarks and leptons, which is given by

$$\begin{aligned} W &= \widehat{Q} (Y_1^d \widehat{H}_1 + Y_3^d \widehat{H}_3 + Y_5^d \widehat{H}_5) \widehat{D}^c + \widehat{L} (Y_1^e \widehat{H}_1 + Y_3^e \widehat{H}_3 + Y_5^e \widehat{H}_5) \widehat{E}^c \\ &+ \widehat{Q} (Y_2^u \widehat{H}_2 + Y_4^u \widehat{H}_4 + Y_6^u \widehat{H}_6) \widehat{U}^c + \widehat{L} (Y_2^\nu \widehat{H}_2 + Y_4^\nu \widehat{H}_4 + Y_6^\nu \widehat{H}_6) \widehat{\nu}^c \\ &+ \mu_{12} \widehat{H}_1 \widehat{H}_2 + \mu_{14} \widehat{H}_1 \widehat{H}_4 + \mu_{16} \widehat{H}_1 \widehat{H}_6 + \mu_{32} \widehat{H}_3 \widehat{H}_2 + \mu_{34} \widehat{H}_3 \widehat{H}_4 \\ &+ \mu_{36} \widehat{H}_3 \widehat{H}_6 + \mu_{52} \widehat{H}_5 \widehat{H}_2 + \mu_{54} \widehat{H}_5 \widehat{H}_4 + \mu_{56} \widehat{H}_5 \widehat{H}_6, \end{aligned} \quad (3.2)$$

where \widehat{Q} and \widehat{L} denote the quark and lepton SU(2)_L doublet superfields, \widehat{U}^c and \widehat{D}^c are quark singlets, and \widehat{E}^c , $\widehat{\nu}^c$, the lepton singlet superfields. The Yukawa matrices Y_i^f can be deduced from eq. (2.5). In what follows, we take the μ_{ij} as effective parameters.

The scalar potential receives the usual contributions from D -, F - and SUSY soft-

breaking terms, which we write below, using for simplicity doublet components.

$$\begin{aligned}
 V_F &= \sum_{\substack{i,j=1,3,5 \\ l=2,4,6}} \mu_{il}^* \mu_{jl} H_i^\dagger H_j + \sum_{\substack{i=1,3,5 \\ k,l=2,4,6}} \mu_{il}^* \mu_{ik} H_k^\dagger H_l, \\
 V_D &= \frac{g^2}{8} \sum_{a=1}^3 \left[\sum_{i=1}^6 H_i^\dagger \tau^a H_i \right]^2 + \frac{g'^2}{8} \left[\sum_{i=1}^6 (-1)^i |H_i|^2 \right]^2, \\
 V_{\text{soft}} &= \sum_{i,j=1,3,5} (m_d^2)_{ij} H_i^\dagger H_j + \sum_{k,l=2,4,6} (m_u^2)_{kl} H_k^\dagger H_l - \sum_{\substack{i=1,3,5 \\ j=2,4,6}} [(B\mu)_{ij} H_i H_j + \text{H.c.}]. \quad (3.3)
 \end{aligned}$$

After electroweak (EW) symmetry breaking, the neutral components of the six Higgs doublets develop VEVs, which we assume to be real,

$$\langle h_{1(3,5)}^0 \rangle = w_{1(3,5)}, \quad \langle h_{2(4,6)}^0 \rangle = w_{2(4,6)}. \quad (3.4)$$

For the purpose of minimising the Higgs potential and computing the tree-level Higgs mass matrices, it proves more convenient to work in the so-called ‘‘Higgs basis’’ [52, 60], where only two of the rotated fields develop VEVs:

$$\phi_i = P_{ij} h_j, \quad (3.5)$$

$$\langle \phi_1^0 \rangle = v_d = \sqrt{w_1^2 + w_3^2 + w_5^2}, \quad \langle \phi_2^0 \rangle = v_u = \sqrt{w_2^2 + w_4^2 + w_6^2}. \quad (3.6)$$

By construction, in order to comply with EW symmetry breaking, the new VEVs must satisfy

$$v_u^2 + v_d^2 \approx (174 \text{ GeV})^2, \quad (3.7)$$

and we can now introduce a generalised definition for $\tan \beta$:

$$\tan \beta = \frac{v_u}{v_d}. \quad (3.8)$$

In the new basis, the free parameters at the EW scale are m_{ij}^2 , b_{ij} , which has dimensions mass^2 , and $\tan \beta$ (for a detailed discussion of the Higgs basis, including the definition of the new parameters and of P_{ij} , we again refer the reader to [55]), and the minimisation equations simply read:

$$\begin{aligned}
 m_{11}^2 &= b_{12} \tan \beta - \frac{M_Z^2}{2} \cos 2\beta, & m_{22}^2 &= b_{12} \cot \beta + \frac{M_Z^2}{2} \cos 2\beta, \\
 m_{13}^2 &= b_{32} \tan \beta, & m_{24}^2 &= b_{14} \cot \beta, \\
 m_{15}^2 &= b_{52} \tan \beta, & m_{26}^2 &= b_{16} \cot \beta.
 \end{aligned} \quad (3.9)$$

The Higgs spectrum derived from the tree-level potential and the previous conditions is far richer than in the usual MSSM case. The neutral sector consists of six CP-even and five CP-odd scalars, while the charged sector will contain ten mass eigenstates. This provides not only a more challenging scenario for potential Higgs detection, but also allows the appearance of dangerous tree-level FCNCs which are severely constrained by observation. The phenomenology related to this multiple-Higgs model and the effects on quark flavour changing transitions have been previously studied in [56]. The impact on lepton flavour violating processes will be discussed in section 4.2.

4. Charged leptons

As mentioned in the Introduction, the previous analysis [56] of the orbifold parameter space has already severely constrained the free parameters of the orbifold. As discussed, we have verified that one could successfully reproduce the observed hierarchy and mixings in the quark sector, and avoid potentially dangerous tree-level FCNCs with a fairly light Higgs boson spectrum. In what follows, we extend our analysis to the lepton sector. In this section we address how reproducing the charged lepton masses further constrains the orbifold parameters, and also discuss possible tree-level lepton flavour violation, arising from the exchange of neutral Higgses.

4.1 Charged lepton masses

We start by considering the mass matrix for the charged leptons, which after FI breaking is given by¹

$$\mathcal{M}^e = g N \epsilon_3 a^L a^{e^c} B^L A^d B^{e^c} = g N \epsilon_3 a^L a^{e^c} \begin{pmatrix} v_1 \epsilon_5^2 \beta^L \beta^{e^c} & v_5 \epsilon_5^2 \beta^L & v_3 \epsilon_5 \alpha^{e^c} \beta^L \\ v_5 \epsilon_5^2 \beta^{e^c} & v_3 & v_1 \alpha^{e^c} \\ v_3 \epsilon_5 \alpha^L \beta^{e^c} & v_1 \alpha^L & v_5 \alpha^L \alpha^{e^c} / \epsilon_5^2 \end{pmatrix}, \quad (4.1)$$

where A^d , a^{L,e^c} and B^{L,e^c} have been defined in eqs. (2.3), (2.6)–(2.8), setting $f = L, e^c$. The next step in the analysis is to determine whether one can find regions of the parameter space where the charged lepton masses can be obtained. We recall that most of the variables appearing in eq. (4.1), namely g , N , ϵ_5 and the Higgs VEVs w_i are also related to the quark sector of the model, and are thus already tightly constrained [56]. We consider the quark input sets studied in ref. [56], used to fix the six Higgs VEVs, and which correctly reproduce the correct mass spectrum for both the up- and down-quarks $\{m_u, m_d, m_c, m_s, m_t, m_b\}$:

$$\text{SET A} = \{0.0040, 0.008, 1.35, 0.130, 180, 4.40\} \text{ GeV}, \quad (4.2)$$

$$\text{SET B} = \{0.0035, 0.008, 1.25, 0.100, 178, 4.50\} \text{ GeV}, \quad (4.3)$$

$$\text{SET C} = \{0.0035, 0.004, 1.15, 0.080, 176, 4.10\} \text{ GeV}, \quad (4.4)$$

$$\text{SET D} = \{0.0040, 0.006, 1.20, 0.105, 178, 4.25\} \text{ GeV}, \quad (4.5)$$

and scan over the ϵ_5 , α^{u^c} and α^{d^c} intervals compatible with realistic quark masses and mixings,

$$0.0085 \leq \epsilon_5 \leq 0.0260, \quad 0.040 \leq \alpha^{u^c} \leq 0.370, \quad 0.190 \leq \alpha^{d^c} \leq 0.842, \quad (4.6)$$

The value of $\tan \beta$, which is also crucial, is tightly related to ϵ_1 [56]. From the previous values, and employing eqs. (2.13), (3.7) and (3.8), we can also derive the value of gN , which

¹Note that the expression for the matrix \mathcal{M}^e in eq. (4.1) corrects the misprint in ref. [49], eq. (69), where the matrix product was taken in the order ABB . A similar correction for neutrino masses will be subsequently taken into account in eqs. (6.1) and (6.11).

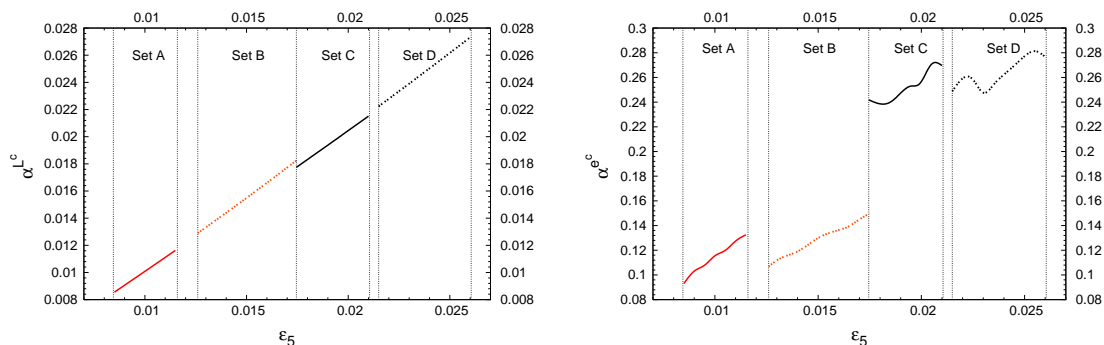


Figure 2: Correlation between the orbifold parameters $(\alpha^{L^c}, \varepsilon_5)$ and $(\alpha^{e^c}, \varepsilon_5)$, for $\tan\beta = 5$ and the distinct sets of input quark masses, A-D (red full lines, red dashed lines, full black and dashed black lines, respectively).

is obtained from the following expression:

$$gN = \frac{1}{a^{u^c}} \frac{(1 + \tan^2\beta)^{1/2}}{\tan\beta} \sqrt{\frac{1}{(\varepsilon_5 \beta^{u^c})^2} \left(m_u + \varepsilon_5^5 \frac{m_t^2}{m_c}\right)^2 + m_c^2 + \left(\frac{m_t \varepsilon_5}{\alpha^{u^c}}\right)^2}. \quad (4.7)$$

Thus, once the several orbifold parameters are determined, one can derive information on the intrinsic orbifold properties, such as the value of the orbifold normalisation constant N , or the heterotic coupling constant g . The numerical analysis of this subsection is instrumental in obtaining the latter information.

Having set the quark parameters and choosing $\tan\beta = 5$ as an example, we proceed to determine ε_3 , α^L , and α^{e^c} . These values will in turn allow to derive the mass matrix for the charged leptons. In agreement with experimental data [1], the latter eigenvalues should be

$$\{m_e, m_\mu, m_\tau\} = \{0.511, 105.41, 1778.45\} \text{ MeV}. \quad (4.8)$$

In figures 2 and 3 we present the values of α^{L,e^c} and a^{L,e^c} giving rise to the correct masses for both the quark and the charged-lepton sectors. The scan over ε_5 has been conducted for the four quark sets in eqs. (4.2)–(4.5). We can see that the behaviour of α^{L,e^c} is completely analogous to what had been observed for the quark sector [56], which is not unexpected, given that the Yukawa couplings for the charged leptons closely follow those of the down-type quarks.

Let us comment on the suppression factor ε_3 . As seen from eq. (4.1), ε_3 is a global factor in the charged-lepton mass matrix. This allows its value to be modified without affecting the mass eigenstates, provided that $\tan\beta$ (i.e. the ratio of the Higgs VEVs) is accordingly changed. In other words, $\tan\beta$ is still an unconstrained degree of freedom, a fact that is particularly useful for the analysis involving the Higgs sector (as discussed in [55]). In figure 4 we display the relation between ε_3 and ε_5 for four different values of $\tan\beta$. It is

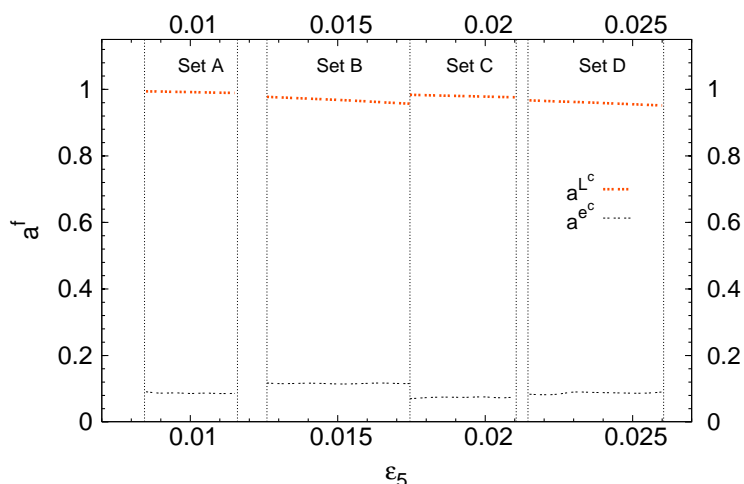


Figure 3: Correlation between the orbifold parameters a^{L^c} , a^{e^c} and ε_5 , for the quark sets A-D and $\tan\beta = 5$.

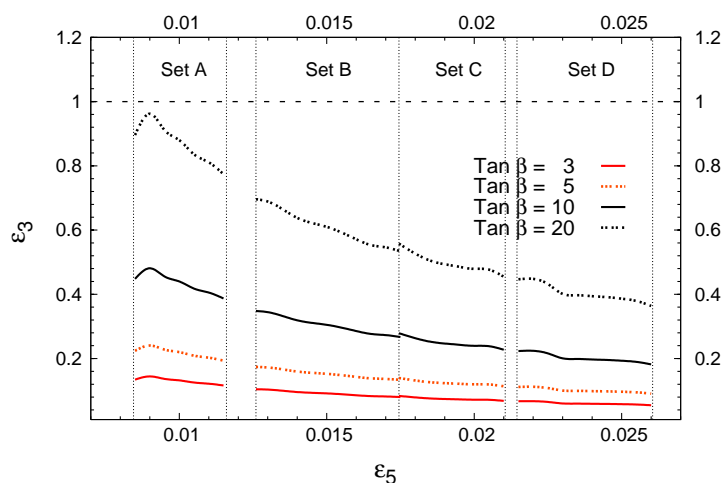


Figure 4: Correlation between ε_3 and ε_5 for different values of $\tan\beta$.

worth mentioning here that, given a particular quark input set, the value of $\tan\beta$ is bounded from above in order to avoid $\varepsilon_3 > 1$. For example, for set A this bound is close to $\tan\beta = 20$. In fact, and as already noticed in the study of the quark sector, the phenomenological viability of these orbifold constructions favours lower values of $\tan\beta$ (not only based on reproducing a viable spectrum, but also related with avoiding excessive FCNCs).

Not only can the quantities ε_i be understood as suppression factors which affect the Yukawa couplings (providing the desired mass hierarchy between fermions), but they are also subject to perturbativity constraints. Concerning the latter, for example ε_3 is actually given by [49]

$$\varepsilon_3 = 3e^{-\frac{2\pi}{3}T_3}(1 + 6e^{-2\pi T_1} + 6e^{-2\pi T_5} + \dots) \approx 3e^{-\frac{2\pi}{3}T_3}, \quad (4.9)$$

where the last approximation corresponds to the assumption of eq. (2.4). Clearly, if ε_i are in general large, T_i have to be small, and therefore perturbativity is spoiled. Under the approximation of eq. (2.4), one can write

$$T_i = -\frac{3}{2\pi} \ln \frac{\varepsilon_i}{3}, \tag{4.10}$$

and, as a consequence, we verify that ε_i cannot be larger than 3, since the T_i VEVs are proportional to R_i^2 , and therefore positive. From the analysis of the orbifold parameters, one can obtain useful information about the high-energy configuration of the string model, namely the size and properties of the compact space, as well as its relation with the gauge unification scale. The allowed regimes for the three T_i and their physical implications will be studied in detail in section 5.

4.2 Tree-level lepton flavour violation

Identical to what occurs for the quark sector, having a scenario with Higgs family replication opens the possibility of tree-level FCNCs in the lepton sector, contrary to what occurs in the SM or in the MSSM. Given the fact that flavour-violating interactions are very suppressed in Nature, one should ensure that the present scenario does not induce excessively large contributions to these processes. In a general multi-Higgs model, it is widely recognised that the most stringent bounds arise from the smallness of the masses of the long- and short-lived neutral kaons. It has been previously verified [56] that for a relatively light Higgs boson spectrum of order $\sim 1 - 5$ TeV, the present orbifold scenario is in very good agreement with experimental data. The analysis was also extended to the B - and D -meson systems, leading to similar bounds for the Higgs masses. With the inclusion of the charged lepton sector in our analysis, it is only natural to expect dangerous lepton flavour-violating interactions. Regarding these interactions, here we have focused on the branching ratios (BRs) of pure leptonic decays of the type $l_i \rightarrow 3l_j$, which have been identified in the literature as the less suppressed processes [53, 54]. In the context of the present orbifold scenario, these decays are going to be generated by Yukawa interactions mediated by neutral Higgs bosons.² As shown in recent studies of LFV in SUSY models with one Higgs family [61 – 63], the one-loop contributions to flavour violating processes can be extremely large for sizable values of $\tan \beta$ and a Higgs mass of order 100-150 GeV. In our case, and as will be shortly confirmed, the requirement that the Higgs bosons are heavy enough to suppress the dangerous quark FCNC interactions indeed ensures that the leptonic processes remain several orders of magnitude below the respective experimental bounds.

In order to study the occurrence of tree-level LFV in the charged-lepton sector we consider the branching ratios of three-body decays, $l_i \rightarrow 3l_j$, mediated by a neutral physical Higgs eigenstate (φ_k). The transition amplitudes and BRs of these processes are then

²We stress here that there are no tree-level contributions to other LFV processes, like radiative decays of the type $l_i \rightarrow l_j \gamma$, which only occur at one-loop level.

given by

$$\Gamma(l_i \rightarrow 3l_j|\varphi_k) = \frac{|\mathcal{Y}_{ji}^k|^2 |\mathcal{Y}_{jj}^k|^2}{128 m_{\varphi_k}^4} \frac{m_{l_i}^5}{192 \pi^3} \quad \text{BR}(l_i \rightarrow 3l_j|\varphi_k) = \frac{1}{128 G_F^2} \frac{|\mathcal{Y}_{ji}^k|^2 |\mathcal{Y}_{jj}^k|^2}{m_{\varphi_k}^4}, \quad (4.11)$$

where m_l is the lepton mass, m_{φ_k} the mass of the mediating scalar/pseudoscalar neutral Higgs, and \mathcal{Y}_{ij}^k is the i, j element of the Yukawa coupling matrix, in the physical mass-eigenstate basis, defined as

$$\mathcal{Y}_{ij}^k = (S^\dagger)_{kl} (V_R^e Y_l^e V_L^{e\dagger})_{ij}. \quad (4.12)$$

In the above, the Higgs physical states are related to the original interaction eigenstates by $\varphi_k = S_{kl} h_l^0$, where h_l^0 are the neutral components of the Higgs doublets (see eq. (3.1)). V_R^e and $V_L^{e\dagger}$ are the matrices which diagonalise the charged-lepton mass matrix, and Y_l^e (with $l = 1, 2, 3$) are the three charged-lepton Yukawa matrices associated to the down-type Higgses, as shown in eq. (3.2). Using the above expressions, we can now compute the contributions of the full Higgs spectrum (six scalars and five pseudoscalars) to the LFV decays. To do so, we choose three distinct Higgs mass textures, already considered in a previous study [56]. Working in the Higgs basis (see section 3) these can be summarily defined via the following parametrisation, which allows to define the Higgs sector via six dimensionless parameters as

$$m_{ij}^{(d)} = \begin{pmatrix} \otimes & \otimes & \otimes \\ \otimes & x_3 & y \\ \otimes & y & x_5 \end{pmatrix} \times 1\text{TeV}, \quad m_{kl}^{(u)} = \begin{pmatrix} \otimes & \otimes & \otimes \\ \otimes & x_4 & y \\ \otimes & y & x_6 \end{pmatrix} \times 1\text{TeV}, \quad \sqrt{b_{ij}} = b \times 1\text{TeV}. \quad (4.13)$$

In the above, $m_{ij}^{d(u)}$ should be understood as the $i, j = 1, 3, 5$ ($k, l = 2, 4, 6$) submatrices of the 6×6 matrix that encodes the rotated soft-breaking Higgs masses in the Higgs basis (see [55]). The symbol \otimes denotes an entry which is fixed by the minima conditions of eq. (3.9). For the Yukawa matrices, we will employ the quark Set A of eq. (4.2) and those values of ε_5 , α^{L^e} and α^{e^c} compatible with realistic masses for the charged leptons (as analysed in section 4). Other sets for the quark masses, B, C or D, will lead to similar results. For simplicity, we will take a near-universality limit for the Higgs-sector textures introduced in eq. (4.13). Regarding the value of $\tan\beta$, and unless otherwise stated, we shall take $\tan\beta = 5$ in the subsequent analysis. We consider the following three cases, with the associated tree-level scalar and pseudoscalar Higgs spectra:

- (1) $x_3 = x_4 = 0.5, x_5 = x_6 = 0.75, y = 0.1, b = 0.1$
 $m^s = \{82.5, 190.6, 493.9, 515.9, 744.4, 760.2\}$ GeV ;
 $m^p = \{186.8, 493.9, 515.9, 744.4, 760.2\}$ GeV .
- (2) $x_3 = x_4 = 0.75, x_5 = x_6 = 1, y = 0.25, b = 0.2$
 $m^s = \{83.6, 292.9, 733.6, 785.9, 987.6, 1057.0\}$ GeV ;
 $m^p = \{291.1, 733.6, 785.9, 987.6, 1057.0\}$ GeV .

LFV process	Present bound	Future sensitivity
$\text{BR}(\mu \rightarrow 3e)$	1.0×10^{-12}	10^{-13}
$\text{BR}(\tau \rightarrow 3e)$	2.0×10^{-7}	10^{-8}
$\text{BR}(\tau \rightarrow 3\mu)$	1.9×10^{-7}	10^{-8}

Table 1: Present bounds and future sensitivities for the LFV processes [64–66].

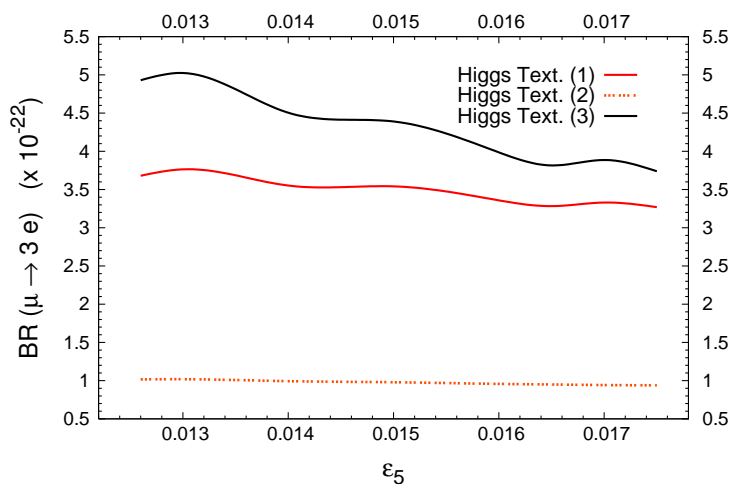


Figure 5: BRs for the Higgs-mediated $\mu \rightarrow 3e$ decay as a function of ϵ_5 , for quark set A and Textures (1-3).

$$\begin{aligned}
 (3) \quad & x_3 = x_4 = 0.5, x_5 = 5, x_6 = 7.5, y = 0.5, b = 0.1 \\
 & m^s = \{82.7, 201.4, 492.4, 516.4, 5000, 7500\} \text{ GeV}; \\
 & m^p = \{197.9, 492.4, 516.4, 5000, 7500\} \text{ GeV}.
 \end{aligned}$$

In the above, m^s and m^p respectively denote the values for the physical scalar and pseudoscalar masses. The results for the decays $\mu \rightarrow 3e$, $\tau \rightarrow 3\mu$ and $\tau \rightarrow 3e$ are summarised in figures 5, 6 and 7. From the latter, we immediately observe that the Higgs-mediated contributions to the $l_i \rightarrow 3l_j$ branching ratios always lie several orders of magnitude below the experimental limits (collected in table 1). This occurs even for Texture (1), associated with a spectrum containing only light (below 1 TeV) Higgs particles. Regarding other relevant LFV processes, as for example leptonic conversion processes in heavy nuclei, which could in principle also receive important tree-level contributions, we have not discussed them here, as these conversion processes are always assumed to be of the same order or even sub-dominant with respect to the leptonic decays (see, for example, [53], [54] or [67]). The extremely low contribution to the purely leptonic decays previously studied (between 5-10 orders of magnitude below the present experimental bounds) renders the impact of these LFV processes clearly negligible, when compared to the flavour-changing processes occurring in the quark sector.

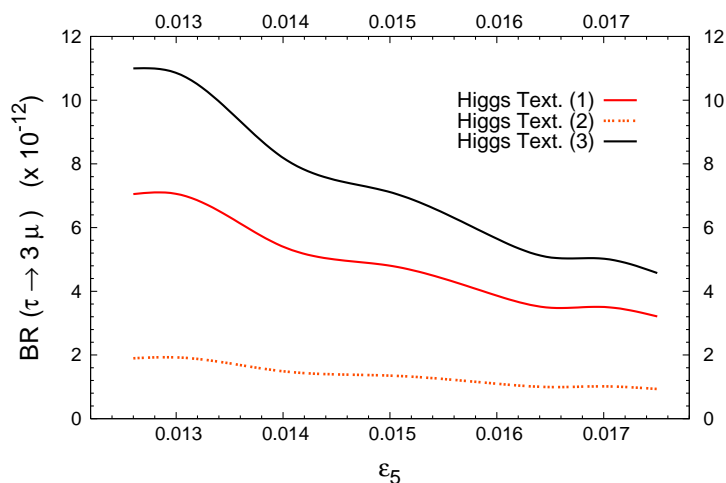


Figure 6: BRs for the Higgs-mediated $\tau \rightarrow 3\mu$ decay as a function of ϵ_5 , for quark set A and Textures (1-3).

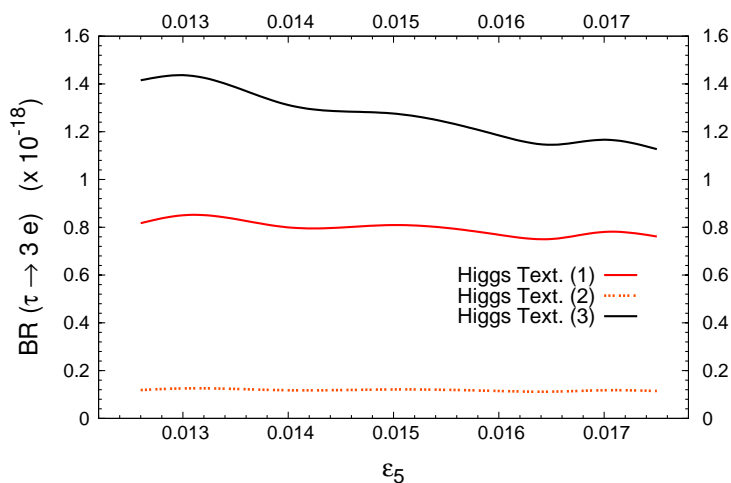


Figure 7: BRs for the Higgs-mediated $\tau \rightarrow 3e$ decay as a function of ϵ_5 , for quark set A and Textures (1-3).

5. Orbifold analysis at the string scale

With the full determination of the quark and charged-lepton sectors we are now ready to address the implications of imposing phenomenological viability at the string level. As previously discussed, the characterisation of the orbifold model is tightly related to the determination of the geometrical suppression factors, ϵ_i , which in turn are instrumental in complying with the different fermion mass hierarchies. As we will discuss in section 6, ϵ_i will further affect the neutrino Yukawa couplings, with an important impact on the seesaw

scale. At this point, it is also relevant to mention that we will not take into account the effect of the renormalisation group equations (RGE) on the quark and lepton mass matrices presented in the previous sections. The flavour structure for the masses is associated with a mechanism taking place at a very high energy scale. However, and given the clearly hierarchical structure of the mass matrices, one does not expect that RGE running will significantly affect the predictions of the model.

In this section, we briefly comment on the information about the shape and size of the compact space, and also discuss the hints on the gauge properties of the string model, which can be inferred from the already constrained values of ε_i .

Let us firstly consider the value of the product of the heterotic coupling constant, g , by the orbifold normalisation constant N , defined in eq. (4.7). The normalisation constant is given by

$$N = \sqrt{V} \frac{3^{3/4}}{8\pi^3} \frac{\Gamma^6(\frac{2}{3})}{\Gamma^3(\frac{1}{3})}. \tag{5.1}$$

In the latter, V denotes the volume of the unit cell of the Z_3 lattice,

$$V = (R_1 R_3 R_5)^2 \left(\sin \frac{2\pi}{3} \right)^3 \tag{5.2}$$

where $R_{1,3,5}$ are the unit cell radii in each sublattice, defined in terms of the three T -moduli as

$$R_i = \frac{4\pi}{3^{1/4}} \sqrt{\text{Re } T_i}. \tag{5.3}$$

From eq. (4.7), taking only the dominant terms into account, we can verify that the assumption of $gN \approx 1$ [49] is indeed valid for values of ε_5 , $\alpha^{u^c} \ll 1$ and $\tan \beta \geq 3$, since

$$gN \approx \frac{\varepsilon_5}{\alpha^{u^c} a^{u^c}} \frac{(1 + \tan^2 \beta)^{1/2}}{\tan \beta} \frac{m_t}{174 \text{ GeV}} \approx \frac{(1 - \varepsilon_5)^{1/2}}{(1 - \alpha^{u^c})^{1/2}} \approx 1, \tag{5.4}$$

where we have used the relation between the orbifold parameters given by eq. (2.11). In figure 8 we display (as presented in ref. [56]) the diagonal moduli T_1 and T_5 for different values of $\tan \beta$. In figure 9 we show the correlation between those values of the moduli T_3 compatible with correct charged-lepton masses and ε_5 , for different values of $\tan \beta$. From all plots we are led to verify that $T_i \sim 1$, implying that the sizes of the radii in each orbifold sublattice are comparable and of order

$$R_i \approx \frac{4\pi}{3^{1/4}} \approx 10. \tag{5.5}$$

As it was previously shown in section 4.1, eq. (4.10), the upper bound for the suppression factors ε_i is 3, above which the moduli are no longer a positive quantity. In the case of T_1 and T_3 , the values are always fixed by the choice of a given $\tan \beta$, as we can see in the previous plots. This in turn implies the existence of an upper limit for $\tan \beta$, above which the moduli become zero. In general, this occurs for values of $\tan \beta$ between 30 and 40, depending on the choice of the remaining orbifold parameters.

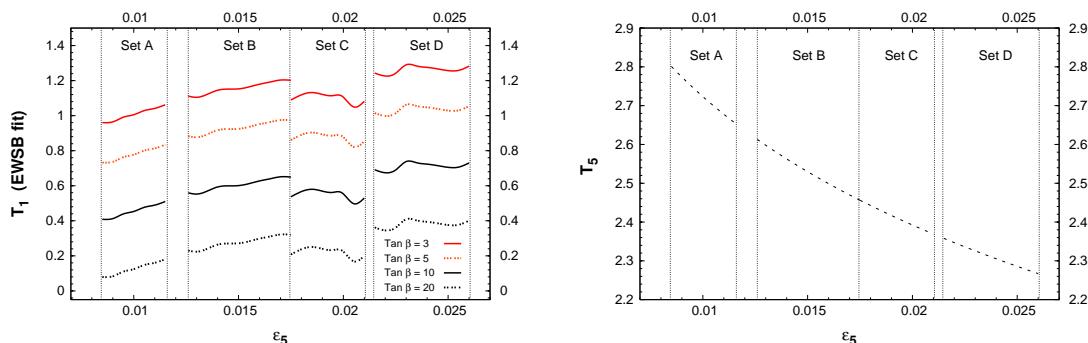


Figure 8: Diagonal lattice moduli, T_1 (left) and T_5 (right), as a function of ϵ_5 [56]. For the case of T_1 , we consider several values of $\tan\beta = 3, 5, 10$ and 20 . For T_5 the dotted line denotes the prediction of the orbifold.

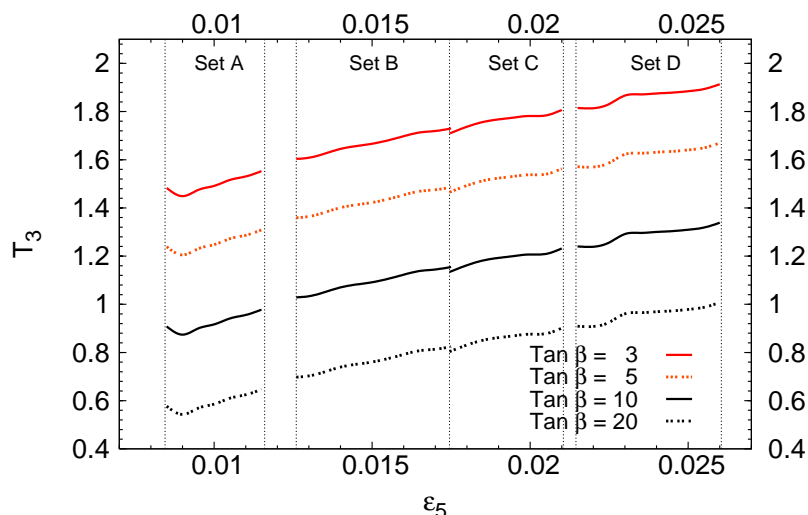


Figure 9: T_3 -moduli dependence on ϵ_5 , for $\tan\beta = 3, 5, 10, 20$.

In the first analysis of ref. [56], the constraints on the orbifold parameters derived from the quark sector had already allow to hint towards a range for the product gN , $1.03 \lesssim gN \lesssim 1.16$. The inclusion of the bounds arising from considering the lepton sector finally allows to refine the knowledge of these orbifold parameters. On the left hand-side of figure 10 we show the value of the orbifold normalisation constant N for different values of $\tan\beta$ as a function of ϵ_5 . From this plot, using eq. (5.4), we can derive the value of the heterotic gauge coupling constant g . The result is presented on the right hand-side of figure 10. As we observe, for the chosen regimes of $\tan\beta$, the value of g varies between $\approx (0.2 - 1.2)$. It is worth noticing here that a value of g of order 1, which is compatible with the above result, was obtained in the orbifold scenarios with three Higgs families analysed

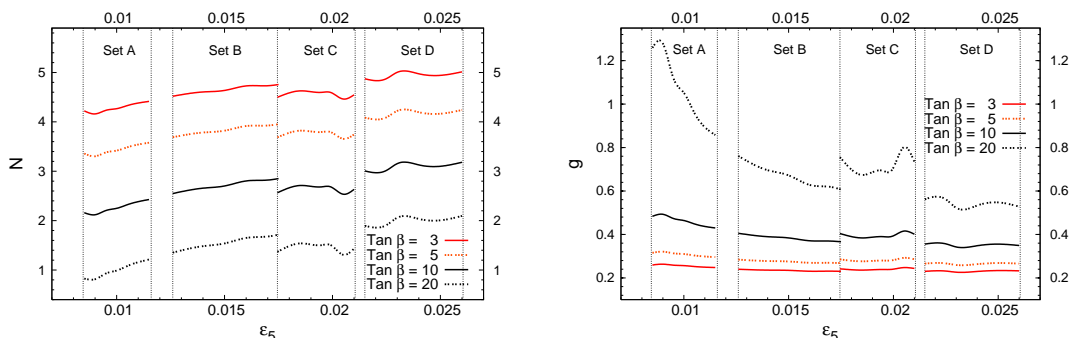


Figure 10: Orbifold normalisation constant N for $\tan \beta = 3, 5, 10, 20$ as a function of ϵ_5 (left), and corresponding values of the heterotic gauge coupling constant g (right).

in [26], in order to solve the discrepancy between the unification scale predicted by the heterotic superstring ($\approx g 5.27 \times 10^{17}$ GeV) and the value deduced from LEP experiments ($\approx 2 \times 10^{16}$ GeV).

6. Neutrinos

As seen from the previous sections, after having imposed the requirements of viable quark masses and mixings, as well as the correct charged lepton masses, many of the orbifold parameters have already been constrained. The question that remains to be answered is whether or not the present neutrino data can be reproduced. In the following subsections, we will discuss how the orbifold scenario allows us to deal with the problem of neutrino masses, providing several mechanisms that can potentially account for an experimentally viable mass spectrum and MNS matrix, including in some of the cases the generation of an effective seesaw mechanism.

6.1 Dirac neutrino masses without seesaw

In the present orbifold scenario, the simplest way of obtaining massive neutrinos is to assume that the latter are Dirac particles, and introduce a Yukawa term, coupling left- and right-handed neutrinos to the up-type Higgs fields. Accordingly, the Dirac mass matrix for the neutral leptons is given by:

$$\mathcal{M}^\nu = g N \epsilon_1 \epsilon_3 a^L a^{\nu c} B^L A^u B^{\nu c} = g N \epsilon_1 \epsilon_3 a^L a^{\nu c} \begin{pmatrix} v_2 \epsilon_5^2 \beta^L \beta^{\nu c} & v_6 \epsilon_5^2 \beta^L & v_4 \epsilon_5 \alpha^{\nu c} \beta^L \\ v_6 \epsilon_5^2 \beta^{\nu c} & v_4 & v_2 \alpha^{\nu c} \\ v_4 \epsilon_5 \alpha^L \beta^{\nu c} & v_2 \alpha^L & v_6 \alpha^L \alpha^{\nu c} / \epsilon_5^2 \end{pmatrix}, \quad (6.1)$$

where A^u , $a^{L,\nu c}$ and $B^{L,\nu c}$ are defined in eqs. (2.3) and (2.6)–(2.8). As shown in [49], unless some fine-tuning is introduced in the scenario the use of these terms without the addition of Majorana couplings gives rise to excessively heavy neutrinos. This can be easily understood by noticing that all the parameters involved in eq. (6.1) are completely determined from

the quark and charged-lepton sectors, the only exception being α^{ν^c} (and thus β^{ν^c} and a^{ν^c}). Thus, the mass eigenvalues of the Dirac neutrinos can be approximately written as:

$$m_{\nu_i} \approx \varepsilon_1 \frac{a^{\nu^c}}{a^{e^c}} m_{l_i}, \tag{6.2}$$

leading to the relation

$$\frac{m_{\nu_i}}{m_{l_i}} \approx \varepsilon_1 \frac{a^{\nu^c}}{a^{e^c}} \sim 10^{-7}. \tag{6.3}$$

Regarding the three parameters appearing in the previous equation, ε_1 is defined by the chosen value of $\tan \beta$ (see [56]). For $\tan \beta$ between 3 and 20, values of ε_1 compatible with realistic quark masses lie in the range $\varepsilon_1 \approx 0.2 - 2$. The factor a^{e^c} has been determined from the charged-lepton sector, $a^{e^c} \approx 0.1$. Thus the remaining free parameter in eq. (6.3) is a^{ν^c} , which depends on α^{ν^c} and ε_5 in the following way (see eq. (2.11)):

$$a^{\nu^c} = \frac{(1 - \alpha^{\nu^c})^{1/2}}{\alpha^{\nu^c}} \frac{\varepsilon_5}{(1 - \varepsilon_5)^{1/2}}. \tag{6.4}$$

Suppression of the light neutrino masses requires values of α^{ν^c} very close to 1, forcing a^{ν^c} to be very small. In turn, this would imply that there are additional fields entering the FI breaking, with a very distinct mass hierarchy (much lighter VEVs), giving rise to terms of the form

$$a^{\nu^c} = \frac{\hat{c}_2^{\nu^c}}{\sqrt{|\hat{c}_1^{\nu^c}|^2 + |\hat{c}_2^{\nu^c}|^2}} \approx 10^{-7} - 10^{-6}. \tag{6.5}$$

To clarify the latter statement, and as an example, let us consider the case in which the factors $\varepsilon'^{(\nu^c)}$ and $\varepsilon''^{(\nu^c)}$, defined in eq. (2.9), are taken to be $\varepsilon'^{(\nu^c)} = 1$ and $\varepsilon''^{(\nu^c)} = \varepsilon_1 \varepsilon_3 \approx 0.01$. In this case, eq. (6.5) may be rewritten as

$$\frac{c_2^{\nu^c}}{c_1^{\nu^c}} \approx 10^{-5} - 10^{-4}. \tag{6.6}$$

In order to fulfil the above condition, we are compelled to modify the original hypothesis of assuming the VEVs c_i^f to be of the order of the FI breaking scale, i.e. 10^{16-17} GeV. One possibility of obtaining the desired hierarchy between $c_1^{\nu^c}$ and $c_2^{\nu^c}$ is to invoke the existence of effective non-renormalisable couplings of the form

$$\frac{\langle C^2 \rangle}{M_{\text{string}}^2} C_2^{\nu^c} \xi_1 \xi_2 \approx 10^{-4} \times C_2^{\nu^c} \xi_1 \xi_2, \tag{6.7}$$

where ξ_1, ξ_2 denote two extra-matter fields which should later mix with the ν^c field [49]. Although this possibility may solve the discrepancy between the FI-breaking scale and the one needed to comply with realistic neutrinos, the introduction of non-renormalisable couplings sets an undesired arbitrariness in the mass scales used to generate the fermion masses. In this sense, it seems preferable to find another way to generate neutrino masses without the addition of higher-order operators.

Another possible solution to this problem could lie in the assumption of a more involved mixing of the fields participating in the FI breaking, as will be presented in section 6.3.

Nevertheless, a more straightforward and simple possibility consists of assuming that the neutrinos are Majorana particles. In this case, one allows the presence of Majorana terms in the superpotential, leading to a type-I seesaw mechanism. There are several possible ways of implementing a seesaw mechanism in the context of these orbifold scenarios, and we pursue this topic in the following subsections.

6.2 Neutrino masses via a type-I seesaw

As first proposed in [49], the introduction of a seesaw mechanism can be easily achieved by considering a Majorana term in the superpotential, arising from the coupling of three extra scalars (of the low-energy spectrum) as follows:

$$W^\nu \sim H^u L \nu^c + S \nu^c \nu^c, \tag{6.8}$$

where S are singlets assigned to the following fixed-point components in the first two sublattices:

$$S \quad \times \quad \times \tag{6.9}$$

Under this assumption, when the singlets develop a VEV, a Majorana mass for the right-handed neutrinos is generated. In the seesaw limit, where the latter VEVs are much heavier than the EW scale, the effective mass matrix for the light neutrinos is then

$$m_\nu^{\text{eff}} \approx \mathcal{M}^\nu (\mathcal{M}^{\nu^c})^{-1} \mathcal{M}^{\nu T}, \tag{6.10}$$

where \mathcal{M}^ν is given in eq. (6.1) and \mathcal{M}^{ν^c} arises from the coupling $S \nu^c \nu^c$, and is thus defined as

$$\mathcal{M}^{\nu^c} = g N a^{\nu^c} a^{\nu^c} B^{\nu^c} A^s B^{\nu^c} \tag{6.11}$$

with

$$A^s = \begin{pmatrix} s_1 & s_3 \varepsilon_5 & s_2 \varepsilon_5 \\ s_3 \varepsilon_5 & s_2 & s_1 \varepsilon_5 \\ s_2 \varepsilon_5 & s_1 \varepsilon_5 & s_3 \end{pmatrix}, \tag{6.12}$$

$s_{1,2,3}$ being the singlet VEVs. Note that in eq. (6.10) the mixing B^{ν^c} cancels, so that the only free parameters in the mass matrix will be the VEVs s_i . It is also important to stress at this point that the Majorana mass term in eq. (6.10) is clearly non-diagonal, with a structure which is determined from the orbifold (analogous to what occurs for all the Dirac mass terms).

The study of the parameter space generated by s_i (for different regimes of the other parameters) reveals that it is possible to generate light neutrino masses of the desired order of magnitude, in good agreement with the experimentally measured mass squared differences between the three species, Δm_{21}^2 and $|\Delta m_{31}^2|$ (see, for example, [2]),

$$\Delta m_{21}^2 = 7.9_{-0.28}^{+0.27} \left({}_{-0.89}^{+1.1} \right) \text{ eV}^2, \tag{6.13}$$

$$|\Delta m_{31}^2| = 2.6 \pm 0.2 (0.6) \times 10^{-3} \text{ eV}^2. \tag{6.14}$$

To illustrate this mechanism, let us define the seesaw mass matrix as it would arise from the following point in the orbifold parameter space, compatible with realistic quark (Set B, eq. (4.3)) and charged-lepton masses,

$$\begin{aligned} \varepsilon_5 &= 0.0126, & \varepsilon_3 &= 0.170, & \varepsilon_1 &= 0.450, \\ \alpha^{u^c} &= 0.089, & \alpha^{d^c} &= 0.295, & \alpha^L &= 0.129. \end{aligned} \quad (6.15)$$

Setting $\tan\beta = 5$, the only remaining parameters in the scenario are the singlet VEVs s_1, s_2, s_3 . The choice of the following values

$$\{s_1, s_2, s_3\} = \{2.45 \times 10^9, 8.89 \times 10^{12}, 1.32 \times 10^{12}\} \text{ GeV}, \quad (6.16)$$

gives us a “normal hierarchy” light neutrino spectrum,

$$\{m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\} = \{5.39 \times 10^{-8}, 9.13 \times 10^{-3}, 5.65 \times 10^{-2}\} \text{ eV}, \quad (6.17)$$

leading to mass squared differences in good agreement with the experimental range of eqs. (6.13), (6.14). Even though this implementation of a type-I seesaw mechanism can lead to a viable light neutrino spectrum, there are two drawbacks to this formulation. The first one comes from the high scale required by the Majorana singlets (10^{9-12} GeV). Again, a possible explanation of this high scale is to assume the fields S_i as effective non-renormalisable FI fields (analogous to the ones suggested in eq. (6.7)) or to allow a more complicated FI mixing which would translate into a further suppression of the Yukawa couplings (see section 6.3, below). The second shortcoming stems from a failure in reproducing the observed mixing in the leptonic sector, as parameterised by the the U_{MNS} matrix

$$U_{\text{MNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & \pm s_{13} \\ -s_{12}c_{23} \mp s_{23}s_{13}c_{12} & c_{12}c_{23} \mp s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{12}s_{23} \mp s_{13}c_{23}c_{12} & -s_{123}c_{12} \mp s_{13}c_{23}c_{12} & c_{23}c_{13} \end{pmatrix}, \quad (6.18)$$

where, for simplicity, we use the CP-conserving parametrisation. The mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$, are experimentally³ given by

$$\theta_{12} = 33.7 \pm 1.3 \begin{pmatrix} +4.3 \\ -3.5 \end{pmatrix}, \quad (6.19)$$

$$\theta_{23} = 43.3 \begin{pmatrix} +4.3 \\ -3.8 \end{pmatrix} \begin{pmatrix} +9.8 \\ -8.8 \end{pmatrix}, \quad (6.20)$$

$$\theta_{13} = 0 \begin{pmatrix} +5.2 \\ -0.0 \end{pmatrix} \begin{pmatrix} +11.5 \\ -0.0 \end{pmatrix}, \quad (6.21)$$

where the angles are expressed in degrees. With the choice of orbifold parameters used in the previous example, eq. (6.15), we find that in this case the mixing angles in the U_{MNS} matrix are

$$\{\theta_{12}^{\text{orbifold}}, \theta_{23}^{\text{orbifold}}, \theta_{13}^{\text{orbifold}}\} = \{48.4, 54.5, 17.9\}. \quad (6.22)$$

³We employ the values given in [2].

As can be verified, the above values lie considerably above the ones allowed by the experimental bounds. The associated U_{MNS} would then be given by

$$U_{\text{MNS}}^{\text{orbifold}} = \begin{pmatrix} 0.998 & 0.045 & 0.022 \\ 0.050 & 0.87 & 0.48 \\ 0.002 & 0.48 & 0.88 \end{pmatrix}. \quad (6.23)$$

This matrix contains nearly the desired mixing for the second and third generations, but fails in reproducing the mixing for the first generation of neutrinos. This behaviour is generic to the surveyed orbifold parameter space, where we have systematically found that no more than two-generation mixing can be satisfied. By varying the singlet VEVs other mixing possibilities can be achieved, but one generation of neutrinos never has a viable mixing with the other two. This appears to be a general feature of the present orbifold scenario, in the sense that it is extremely difficult to simultaneously accommodate the observed near-maximal mixing in the lepton sector and the small one evidenced in the CKM matrix.

A second possibility of implementing a type-I seesaw, without the need of considering a more intricate FI breaking, consists in assuming the existence of an intermediate scale. In principle this scale is not predicted by the orbifold formulation, but it would nevertheless allow to accommodate the experimental data in view of orbifold-derived neutrino Yukawa couplings. In particular, in this case one is allowing for additional sources of unconstrained mixing in the lepton sector, stemming from heavy Majorana neutrino interactions. Thus the effective light neutrino mass matrix is obtained from the seesaw equation, and given by

$$m_\nu = \mathcal{M}^\nu (M_R)^{-1} \mathcal{M}^{\nu T}, \quad (6.24)$$

$$U_{\text{MNS}}^T m_\nu U_{\text{MNS}} = m_\nu^{\text{diag}}, \quad (6.25)$$

where \mathcal{M}^ν is defined in eq. (6.1) and M_R is the Majorana mass matrix, whose values are not determined by orbifold considerations. In general, U_{MNS} , m_ν^{diag} are known and a very simple structure is adopted for M_R (namely a diagonal matrix) in order to derive the unknown Yukawa couplings. In the present approach, we do know the Yukawa couplings (from the orbifold construction, which at this stage has become strongly constrained), and phenomenological viability of the orbifold scenario indirectly suggests the structure of M_R . Noticing that the seesaw equation can be rewritten as

$$\mathcal{M}^\nu U_{\text{MNS}} (m_\nu^{\text{diag}})^{-1} U_{\text{MNS}}^T \mathcal{M}^{\nu T} = M_R, \quad (6.26)$$

we obtain M_R as required to comply with data on neutrino masses and mixings. It is important to notice that we are not working in a basis where the charged lepton Yukawa couplings are diagonal, implying that the U_{MNS} matrix is defined as

$$U_{\text{MNS}} = V_L^{E\dagger} U^n, \quad (6.27)$$

where V_L^E is the unitary matrix that rotates the left-handed charged lepton fields, so to diagonalise \mathcal{M}^e , while U^n is the matrix that diagonalises the symmetric neutrino mass matrix, m_ν .

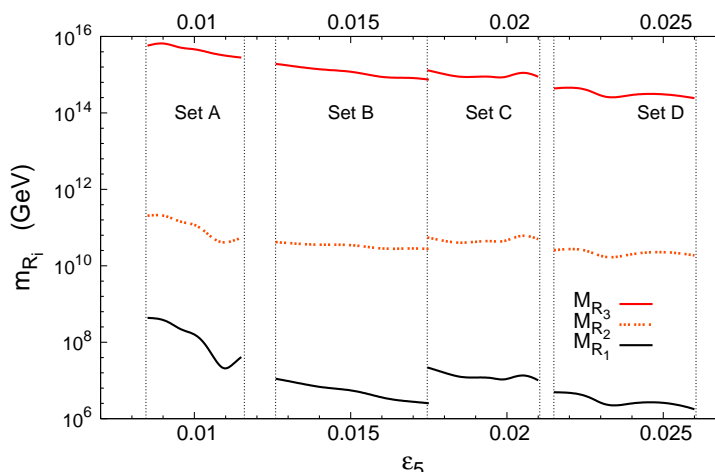


Figure 11: Eigenvalues of M_R (in GeV) as a function of ε_5 , for the input orbifold parameter sets A, B, C and D. We take $\tan\beta = 5$, $\theta_{13} = 8^\circ$, and $m_{\nu_i} = \{10^{-5}, 0.0089, 0.0509\}$ eV.

In figure 11, we depict the eigenvalues of M_R , as a function of ε_5 , for the input orbifold parameter sets A, B, C and D. Leading to this figure we have assumed $\tan\beta = 5$, $\theta_{13} = 8^\circ$, a regime of $\alpha^{\nu^c} \approx \varepsilon_5$, and a normal hierarchy for the light neutrino spectrum, namely

$$m_{\nu_i} = \{10^{-5}, 0.0089, 0.0509\} \text{ eV} . \quad (6.28)$$

As can be seen from figure 11, the orbifold structure would indeed suggest the existence of heavy Majorana neutrinos, whose masses would lie in the $10^6 - 10^8$ GeV and $10^{14} - 10^{16}$ GeV ranges for the lightest and heaviest states, respectively. Naturally, the heavy spectrum strongly reflects the input parameters, with the most important role being played by α^{ν^c} , $\tan\beta$ and the hierarchy of the light neutrinos. Essentially $\tan\beta$ translates in an overall factor, and having normal/inverted hierarchy or quasi-degenerate light neutrinos mostly affects the m_{R_i} pattern. On the other hand, the chosen α^{ν^c} range can have a crucial impact: while values of $\alpha^{\nu^c} \approx \varepsilon_5$ (as used for figure 11) lead to m_{R_1} masses in the $10^6 - 10^8$ GeV range, larger values, close to 1, can even give rise to masses as small as $\mathcal{O}(\text{TeV})$. The phenomenological implications of the latter regime would be extensive, and we do not address them here.

Finally, and as an illustrative example, we present the complete M_R matrix structure, for the orbifold set of parameters taken in eq. (6.15), $\tan\beta = 5$, $\theta_{13} = 1^\circ$, $\alpha^{\nu^c} = \varepsilon_5$ and the light-neutrino spectrum of eq. (6.28):

$$M_R = \begin{pmatrix} 3.86 \times 10^{11} & 1.73 \times 10^{12} & -3.35 \times 10^{13} \\ 1.73 \times 10^{12} & 7.73 \times 10^{12} & -1.50 \times 10^{14} \\ -3.35 \times 10^{13} & -1.50 \times 10^{14} & 2.92 \times 10^{15} \end{pmatrix} \text{ GeV} . \quad (6.29)$$

Using this matrix one can check that both a high seesaw scale and additional mixings involving the right-handed neutrinos should be invoked in order to reproduce the correct

neutrino masses and mixings. The eigenvalues for the Majorana mass matrix (which correspond to the masses of the heavy neutrinos) are

$$\{m_{R_1}, m_{R_2}, m_{R_3}\} = \{1.43 \times 10^7, 2.12 \times 10^{10}, 2.93 \times 10^{15}\} \text{ GeV}, \quad (6.30)$$

From these values one can see that, due to the mixing in the upper-left 2×2 block matrix in eq. (6.29), we encounter a very suppressed mass eigenvalue. For different regimes in the relevant parameters considered ($\tan \beta$, θ_{13}^{MNS} , α^{ν^c} and the light-neutrino mass spectrum) one can check that this eigenvalue may be even sufficiently small to lie at the TeV scale, being thus potentially detectable. The other eigenvalues, as can be seen in figure 11, remain always heavy, between 10^{10} and 10^{16} GeV.

Although phenomenological viable, these last implementation of a type-I seesaw is, as previously mentioned, neither related to the geometry of the orbifold, nor to the dynamics associated with FI breaking. In what follows, we pursue one final avenue, possibly leading to a more appealing seesaw realisation.

6.3 A viable seesaw from the FI breaking

As mentioned, a third possibility for reproducing the observed neutrino masses and mixings may be related with assuming a more complex FI breaking. Here, we briefly outline the idea, for the simplest case of one generation. Let us then assume that in addition to the L and ν^c fields, which have the standard location

$$L \leftrightarrow \bullet \bullet \quad \nu^c \leftrightarrow \times \times, \quad (6.31)$$

there are additional matter fields (triplets, doublet or singlets) ζ_i , coupled to the C_i fields which develop very large VEVs, thus inducing FI breaking. One can assume that these fields have the following assignments with respect to the first two sublattices:

$$\begin{array}{lll} \zeta_1 \leftrightarrow \bullet \bullet & \zeta_2 \leftrightarrow \times \circ & \zeta_3 \leftrightarrow \bullet \times \\ C_1 \leftrightarrow \bullet \bullet & C_2 \leftrightarrow \circ \times & C_3 \leftrightarrow \bullet \times \end{array} \quad (6.32)$$

The latter C_i develop VEVs, $c_i = \langle C_i \rangle$, with $c_i \approx \mathcal{O}(10^{16-17})$ GeV. In principle, one can also have the following terms in the superpotential

$$C_1 \zeta_1 L + C_2 \zeta_2 \zeta_1 + C_2 \nu^c \zeta_3 + C_3 \zeta_3 \zeta_3. \quad (6.33)$$

With the above proposed lattice assignments, and after FI breaking, this would lead to

$$c_1 \zeta_1 L + \varepsilon_1 \varepsilon_3 c_2 \zeta_2 \zeta_1 + \varepsilon_1 c_2 \nu^c \zeta_3 + c_3 \zeta_3 \zeta_3. \quad (6.34)$$

In the basis defined by $(L \nu^c \zeta_1 \zeta_2 \zeta_3)^T$, one would then arrive at the following ‘‘mass matrix’’ (again neglecting family dependence as a first approach),

$$M_{\text{FI}}^\nu = \begin{pmatrix} 0 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_1 c_2 \\ c_1 & 0 & 0 & \varepsilon_1 \varepsilon_3 c_2 & 0 \\ 0 & 0 & \varepsilon_1 \varepsilon_3 c_2 & 0 & 0 \\ 0 & \varepsilon_1 c_2 & 0 & 0 & c_3 \end{pmatrix}, \quad (6.35)$$

with eigenvalues given by

$$m_i^{\text{FI}} = \left\{ 0, \pm \sqrt{c_1^2 + \varepsilon_1^2 \varepsilon_3^2 c_2^2}, \frac{1}{2} \left(c_3 \pm \sqrt{c_3^2 + 4 \varepsilon_1^2 c_2^2} \right) \right\}. \quad (6.36)$$

Further assuming that we are in the limit where $\tan \beta$ is low (favoured from several arguments, as discussed throughout this work), ε_1 is smaller than unity. In this limit, and given that $c_2 \sim c_3$, one would find

$$m_4 \approx c_3 + \varepsilon_1^2 c_3, \quad m_5 \approx -\varepsilon_1^2 c_3, \quad (6.37)$$

thus implying the presence of a term in the superpotential behaving like

$$-\varepsilon_1^2 c_3 \nu^{c'} \nu^{c'}, \quad (6.38)$$

with

$$\nu^{c'} \propto \left(-c_3 + \sqrt{c_3^2 + 4 \varepsilon_1^2 c_2^2} \right) \nu^c + (2 \varepsilon_1 c_2) \zeta_3. \quad (6.39)$$

In the superpotential involving the MSSM fields, the term in eq. (6.38) would effectively generate a Majorana mass term for the neutrino field. Thus, an intermediate Majorana scale (lower than the FI breaking scale, and much heavier than the EW scale) would naturally appear, induced from the dynamics of FI breaking. Other mixings between ζ_i and the remaining matter fields could in principle occur, but can be suppressed by some appropriate symmetry.

Whether or not such a FI-seesaw would indeed reproduce the correct three family neutrino masses and mixings is a question worth discussion. Additionally, one should also recall that the existence of heavy Majorana neutrinos, with possibly complex Yukawa couplings (for a discussion of how to implement CP violation in this class of orbifold constructions, see ref. [56]), offers the possibility of generating the observed baryon asymmetry of the Universe from thermal leptogenesis [68]. This can be an involved question, given that as seen in section 6.2, the scale of the lightest right-handed neutrino can range over several orders of magnitude. These two issues, and others like the collider signatures of potentially light Majorana neutrinos will be the subject of a subsequent analysis.

7. Conclusions

In this study we have aimed at completing the analysis of the phenomenological viability of abelian Z_3 orbifold compactifications with two Wilson lines. This class of models, which naturally includes three families of fermions and Higgs fields, offers the possibility of obtaining realistic fermion masses and mixings, entirely at the renormalisable level. The Yukawa couplings arise from the geometrical configuration of the orbifold, and since they are explicitly calculable, one can explore possible solutions to the flavour problem of the SM and MSSM.

Our analysis here has been a phenomenological “bottom-up” one [49]. That is the particles were assigned to fixed points in a way that can reproduce the experimental data.

In addition the scheme relies on the mixing between fields due to the FI breaking. We have not completed a full analysis of minimizing the potential along D- and F-flat directions after the breaking, but we have made use of the very general features that such a minimization should have, namely mixing of the would-be MSSM fields with other singlet, doublet and triplet fields that couple to heavy fields [16, 17, 12]. Under these assumptions, successfully reproducing the observed pattern of quark masses and mixings already severely constrains the orbifold parameters [56]. Furthermore, the presence of six Higgs doublets poses potential problems regarding tree-level FCNCs, which can nevertheless be avoided with a fairly light Higgs boson spectrum.

We have addressed in detail the implications of this class of orbifold compactifications for the lepton sector. Regarding the charged leptons, we have to use for the analysis the still unconstrained orbifold parameters. We have verified that for reasonable values of these we can easily account for the observed spectrum. Moreover, and even though one is equally likely to encounter tree-level contributions to three-body LFV decays, the typical choices of Higgs soft-breaking masses (taken as to comply with the bounds on neutral meson FCNC) ensure that the predicted BRs lie several orders of magnitude below the experimental bounds.

Regarding the neutrino sector, after having imposed the requirements of viable quark masses and mixings, as well as the correct charged lepton masses, many of the orbifold parameters have already been constrained. Nevertheless, the orbifold scenarios still offer several possibilities. Albeit promising, we verified that the hypothesis of strictly Dirac neutrinos requires that the fields entering the FI breaking should have extremely hierarchical VEVs, forcing to call upon effective non-renormalisable couplings. Implementing a type-I seesaw mechanism via extra singlet fields whose interactions are dictated by the orbifold configuration reveals to be equally difficult. Complying with the measured mass squared differences favours VEVs for the Majorana singlets far higher than for the other fields. This again introduced the need to interpret these fields as effective non-renormalisable fields. Additionally, this mechanism fails in accommodating the current bounds on the neutrino mixing angles.

The need of additional mixing involving the Majorana singlet sector, and of an intermediate scale of about 10^{9-10} GeV motivated us to consider a third possibility. We have thus assumed that the smallness of the light neutrino masses is indeed explained by a type-I seesaw mechanism, where nor the scale, nor the mixings of the heavy singlets are predicted by the orbifold. In this case we verified that neutrino masses and mixings can be easily obtained, with a particularly interesting possibility which is that of a TeV-mass Majorana singlet.

In spite of the latter, it would be theoretically more appealing and consistent to have neutrino masses and mixings strictly from geometrical argumentations and/or from FI breaking. We pursued this challenging possibility, finding that in the simplest one-generation case, a more involved FI breaking can in principle give rise to a Majorana mass term, with a scale far lower than that of FI breaking, and much higher than the EW scale.

This final possibility is definitely worth further investigation. In addition, one can also investigate the viability of generating the observed baryon asymmetry of the Universe from

thermal leptogenesis. Having Majorana singlets that can be (although not necessarily) as light as the EW scale, also poses interesting scenarios regarding collider signatures.

Acknowledgments

N. Escudero is deeply grateful to the members of the Laboratoire de Physique Théorique, Université de Paris-Sud XI, for their kind hospitality in Orsay during the final stages of this work. He also thanks E. Arganda for useful discussions.

The work of N. Escudero is supported by the “Consejería de Educación de la Comunidad de Madrid - FPI Program” and “European Social Fund”. The work of C. Muñoz is supported in part by the Spanish DGI of the MEC under Proyectos Nacionales FPA2006-05423 and FPA2006-01105, by the European Union under the RTN program MRTN-CT-2004-503369, and by the Comunidad de Madrid under Proyecto HEPHACOS, Ayudas de I+D S-0505/ESP-0346. The work of A. M. Teixeira is supported by the French ANR project PHYS@COL&COS.

References

- [1] PARTICLE DATA GROUP collaboration, S. Eidelman et al., *Review of particle physics*, *Phys. Lett.* **B 592** (2004) 1.
- [2] M.C. Gonzalez-Garcia and M. Maltoni, *Phenomenology with massive neutrinos*, [arXiv:0704.1800](https://arxiv.org/abs/0704.1800).
- [3] M. Maltoni, T. Schwetz, M.A. Tortola and J.W.F. Valle, *Status of global fits to neutrino oscillations*, *New J. Phys.* **6** (2004) 122 [[hep-ph/0405172](https://arxiv.org/abs/hep-ph/0405172)].
- [4] A. Strumia and F. Vissani, *Implications of neutrino data circa 2005*, *Nucl. Phys.* **B 726** (2005) 294 [[hep-ph/0503246](https://arxiv.org/abs/hep-ph/0503246)].
- [5] G.L. Fogli, E. Lisi, A. Marrone and A. Palazzo, *Global analysis of three-flavor neutrino masses and mixings*, *Prog. Part. Nucl. Phys.* **57** (2006) 742 [[hep-ph/0506083](https://arxiv.org/abs/hep-ph/0506083)].
- [6] D.J. Gross, J.A. Harvey, E.J. Martinec and R. Rohm, *The heterotic string*, *Phys. Rev. Lett.* **54** (1985) 502; *Heterotic string theory. 1. The free heterotic string*, *Nucl. Phys.* **B 256** (1985) 253; *Heterotic string theory. 2. The interacting heterotic string*, *Nucl. Phys.* **B 267** (1986) 75.
- [7] L.J. Dixon, J.A. Harvey, C. Vafa and E. Witten, *Strings on orbifolds*, *Nucl. Phys.* **B 261** (1985) 678; *Strings on orbifolds. 2*, *Nucl. Phys.* **B 274** (1986) 285.
- [8] L.E. Ibáñez, H.P. Nilles and F. Quevedo, *Orbifolds and Wilson lines*, *Phys. Lett.* **B 187** (1987) 25.
- [9] L.E. Ibáñez, J.E. Kim, H.P. Nilles and F. Quevedo, *Orbifold compactifications with three families of $SU(3) \times SU(2) \times U(1)^n$* , *Phys. Lett.* **B 191** (1987) 282.
- [10] D. Bailin, A. Love and S. Thomas, *A three generation orbifold compactified superstring model with realistic gauge group*, *Phys. Lett.* **B 194** (1987) 385.
- [11] L.E. Ibáñez, J. Mas, H.-P. Nilles and F. Quevedo, *Heterotic strings in symmetric and asymmetric orbifold backgrounds*, *Nucl. Phys.* **B 301** (1988) 157.

- [12] J.A. Casas, E.K. Katehou and C. Muñoz, *U(1) charges in orbifolds: anomaly cancellation and phenomenological consequences*, *Nucl. Phys. B* **317** (1989) 171.
- [13] A. Font, L.E. Ibáñez, H.P. Nilles and F. Quevedo, *Degenerate orbifolds*, *Nucl. Phys. B* **307** (1988) 109 [*Erratum ibid.* **B 310** (1988) 764].
- [14] J.E. Kim, *The strong CP problem in orbifold compactifications and an $SU(3) \times SU(2) U(1)^n$* , *Phys. Lett. B* **207** (1988) 434.
- [15] J.A. Casas and C. Muñoz, *Three generation $SU(3) \times SU(2) \times U(1)_Y \times U(1)$ orbifold models through Fayet-Iliopoulos terms*, *Phys. Lett. B* **209** (1988) 214.
- [16] J.A. Casas and C. Muñoz, *Three generation $SU(3) \times SU(2) \times U(1)_Y$ models from orbifolds*, *Phys. Lett. B* **214** (1988) 63.
- [17] A. Font, L.E. Ibáñez, H.P. Nilles and F. Quevedo, *Yukawa couplings in degenerate orbifolds: towards a realistic $SU(3) \times SU(2) \times U(1)$ superstring*, *Phys. Lett. B* **210** (1988) 101 [*Erratum ibid.* **B 213** (1988) 564].
- [18] J.A. Casas and C. Muñoz, *Yukawa couplings in $SU(3) \times SU(2) \times U(1)_Y$ orbifold models*, *Phys. Lett. B* **212** (1988) 343.
- [19] J.A. Casas and C. Muñoz, *Restrictions on realistic superstring models from renormalization group equations*, *Phys. Lett. B* **214** (1988) 543.
- [20] A. Font, L.E. Ibáñez, F. Quevedo and A. Sierra, *The construction of 'realistic' four-dimensional strings through orbifolds*, *Nucl. Phys. B* **331** (1990) 421.
- [21] J.A. Casas, M. Mondragon and C. Muñoz, *Reducing the number of candidates to standard model in the Z_3 orbifold*, *Phys. Lett. B* **230** (1989) 63.
- [22] Y. Katsuki et al., *Z_n orbifold models*, *Nucl. Phys. B* **341** (1990) 611.
- [23] H.B. Kim and J.E. Kim, *An orbifold compactification with three families from twisted sectors*, *Phys. Lett. B* **300** (1993) 343 [[hep-ph/9212311](#)].
- [24] G. Aldazabal, A. Font, L.E. Ibáñez and A.M. Uranga, *Building guts from strings*, *Nucl. Phys. B* **465** (1996) 34 [[hep-th/9508033](#)].
- [25] J. Giedt, *Spectra in standard-like Z_3 orbifold models*, *Ann. Phys. (NY)* **297** (2002) 67 [[hep-th/0108244](#)].
- [26] C. Muñoz, *A kind of prediction from superstring model building*, *JHEP* **12** (2001) 015 [[hep-ph/0110381](#)].
- [27] T. Kobayashi, S. Raby and R.-J. Zhang, *Searching for realistic 4D string models with a Pati-Salam symmetry: orbifold grand unified theories from heterotic string compactification on a Z_6 orbifold*, *Nucl. Phys. B* **704** (2005) 3 [[hep-ph/0409098](#)].
- [28] J. Giedt, G.L. Kane, P. Langacker and B.D. Nelson, *Massive neutrinos and (heterotic) string theory*, *Phys. Rev. D* **71** (2005) 115013 [[hep-th/0502032](#)].
- [29] T. Kobayashi and C. Muñoz, *More about soft terms and FCNC in realistic string constructions*, *JHEP* **01** (2006) 044 [[hep-ph/0508286](#)].
- [30] W. Buchmüller, K. Hamaguchi, O. Lebedev and M. Ratz, *Supersymmetric standard model from the heterotic string*, *Phys. Rev. Lett.* **96** (2006) 121602 [[hep-ph/0511035](#)];
W. Buchmüller, K. Hamaguchi, O. Lebedev and M. Ratz, *Supersymmetric standard model from the heterotic string. II*, *Nucl. Phys. B* **785** (2007) 149 [[hep-th/0606187](#)].

- [31] J.E. Kim and B. Kyae, *String MSSM through flipped SU(5) from Z_{12} orbifold*, hep-th/0608085; *Flipped SU(5) from Z_{12-I} orbifold with Wilson line*, Nucl. Phys. **B 770** (2007) 47 [hep-th/0608086].
- [32] O. Lebedev et al., *A mini-landscape of exact MSSM spectra in heterotic orbifolds*, Phys. Lett. **B 645** (2007) 88 [hep-th/0611095]; *Low energy supersymmetry from the heterotic landscape*, Phys. Rev. Lett. **98** (2007) 181602 [hep-th/0611203]; *The heterotic road to the MSSM with R parity*, arXiv:0708.2691.
- [33] I.-W. Kim, J.E. Kim and B. Kyae, *Harmless R-parity violation from Z_{12-I} compactification of $E_8 \times E'_8$ heterotic string*, Phys. Lett. **B 647** (2007) 275 [hep-ph/0612365].
- [34] J.E. Kim, J.-H. Kim and B. Kyae, *Superstring standard model from Z_{12-I} orbifold compactification with and without exotics and effective R-parity*, JHEP **06** (2007) 034 [hep-ph/0702278].
- [35] J.M. Drummond, G. Feverati, L. Frappat and É. Ragoucy, *Super-Hubbard models and applications*, JHEP **05** (2007) 008 [hep-th/0703078].
- [36] E. Witten, *Some properties of $O(32)$ superstrings*, Phys. Lett. **B 149** (1984) 351.
- [37] M. Dine, N. Seiberg and E. Witten, *Fayet-Iliopoulos terms in string theory*, Nucl. Phys. **B 289** (1987) 589.
- [38] J.J. Atick, L.J. Dixon and A. Sen, *String calculation of Fayet-Iliopoulos d terms in arbitrary supersymmetric compactifications*, Nucl. Phys. **B 292** (1987) 109.
- [39] M. Dine, I. Ichinose and N. Seiberg, *F terms and d terms in string theory*, Nucl. Phys. **B 293** (1987) 253.
- [40] S. Hamidi and C. Vafa, *Interactions on orbifolds*, Nucl. Phys. **B 279** (1987) 465.
- [41] L.J. Dixon, D. Friedan, E.J. Martinec and S.H. Shenker, *The conformal field theory of orbifolds*, Nucl. Phys. **B 282** (1987) 13.
- [42] L.E. Ibáñez, *Hierarchy of quark-lepton masses in orbifold superstring compactification*, Phys. Lett. **B 181** (1986) 269.
- [43] J.A. Casas and C. Muñoz, *Fermion masses and mixing angles: a test for string vacua*, Nucl. Phys. **B 332** (1990) 189 [Erratum *ibid.* **B 340** (1990) 280].
- [44] J.A. Casas, F. Gómez and C. Muñoz, *Fitting the quark and lepton masses in string theories*, Phys. Lett. **B 292** (1992) 42 [hep-th/9206083].
- [45] J.A. Casas, F. Gómez and C. Muñoz, *World sheet instanton contribution to Z_7 Yukawa couplings*, Phys. Lett. **B 251** (1990) 99.
- [46] T.T. Burwick, R.K. Kaiser and H.F. Muller, *General Yukawa couplings of strings on Z_N orbifolds*, Nucl. Phys. **B 355** (1991) 689.
- [47] T. Kobayashi and N. Ohtsubo, *Geometrical aspects of Z_N orbifold phenomenology*, Int. J. Mod. Phys. **A 9** (1994) 87.
- [48] J.A. Casas, F. Gómez and C. Muñoz, *Complete structure of Z_N Yukawa couplings*, Int. J. Mod. Phys. **A 8** (1993) 455 [hep-th/9110060].
- [49] S.A. Abel and C. Muñoz, *Quark and lepton masses and mixing angles from superstring constructions*, JHEP **02** (2003) 010 [hep-ph/0212258].

- [50] P. Ko, T. Kobayashi and J.-h. Park, *Quark masses and mixing angles in heterotic orbifold models*, *Phys. Lett.* **B 598** (2004) 263 [[hep-ph/0406041](#)].
- [51] P. Ko, T. Kobayashi and J.-h. Park, *Lepton masses and mixing angles from heterotic orbifold models*, *Phys. Rev.* **D 71** (2005) 095010 [[hep-ph/0503029](#)].
- [52] H. Georgi and D.V. Nanopoulos, *Suppression of flavor changing effects from neutral spinless meson exchange in gauge theories*, *Phys. Lett.* **B 82** (1979) 95.
- [53] B. McWilliams and L.-F. Li, *Virtual effects of Higgs particles*, *Nucl. Phys.* **B 179** (1981) 62.
- [54] O. Shanker, *Flavor violation, scalar particles and leptiquarks*, *Nucl. Phys.* **B 206** (1982) 253.
- [55] N. Escudero, C. Muñoz and A.M. Teixeira, *Fcncs in supersymmetric multi-Higgs doublet models*, *Phys. Rev.* **D 73** (2006) 055015 [[hep-ph/0512046](#)].
- [56] N. Escudero, C. Muñoz and A.M. Teixeira, *Phenomenological viability of orbifold models with three Higgs families*, *JHEP* **07** (2006) 041 [[hep-ph/0512301](#)].
- [57] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870.
- [58] B. Pontecorvo, *Mesonium and antimesonium*, *Sov. Phys. JETP* **6** (1957) 429.
- [59] P. Minkowski, $\mu \rightarrow e\gamma$ at a rate of one out of 1-billion muon decays?, *Phys. Lett.* **B 67** (1977) 421;
M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories*, P. Van. Nieuwenhuizen and D.Z. Freedman eds., in *Supergravity*, North-Holland, Amsterdam (1979) [Print-80-0576 (CERN)];
T. Yanagida, *Horizontal symmetry and masses of neutrinos*, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, O. Sawada and A. Sugamoto eds., KEK, Tsukuba (1979);
S.L. Glashow, *Quarks and Leptons*, M. Lévy et al. eds., Plenum Press, New York (1980);
R.N. Mohapatra and G. Senjanović, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912;
J. Schechter and J.W.F. Valle, *Neutrino masses in $SU(2) \times U(1)$ theories*, *Phys. Rev.* **D 22** (1980) 2227;
J. Schechter and J.W.F. Valle, *Neutrino decay and spontaneous violation of lepton number*, *Phys. Rev.* **D 25** (1982) 774.
- [60] M. Drees, *Supersymmetric models with extended Higgs sector*, *Int. J. Mod. Phys.* **A 4** (1989) 3635.
- [61] E. Arganda and M.J. Herrero, *Testing supersymmetry with lepton flavor violating tau and mu decays*, *Phys. Rev.* **D 73** (2006) 055003 [[hep-ph/0510405](#)].
- [62] S. Antusch, E. Arganda, M.J. Herrero and A.M. Teixeira, *Impact of θ_{13} on lepton flavour violating processes within SUSY seesaw*, *JHEP* **11** (2006) 090 [[hep-ph/0607263](#)].
- [63] E. Arganda, M.J. Herrero and A.M. Teixeira, $\mu - e$ conversion in nuclei within the CMSSM seesaw: universality versus non-universality, *JHEP* **10** (2007) 104 [[arXiv:0707.2955](#)].
- [64] SINDRUM collaboration, U. Bellgardt et al., *Search for the decay $\mu^+ \rightarrow e^+e^+e^-$* , *Nucl. Phys.* **B 299** (1988) 1.
- [65] BABAR collaboration, B. Aubert et al., *Search for lepton flavor violation in the decay $\tau^- \rightarrow \ell^- \ell^+ \ell^-$* , *Phys. Rev. Lett.* **92** (2004) 121801 [[hep-ex/0312027](#)].

- [66] SUPERKEKB PHYSICS WORKING GROUP collaboration, A.G. Akeroyd et al., *Physics at super B factory*, [hep-ex/0406071](#).
- [67] D. Ng and J.N. Ng, *Can $\mu - e$ conversion in nuclei be a good probe for lepton number violating Higgs couplings?*, *Phys. Lett. B* **320** (1994) 181 [[hep-ph/9308352](#)].
- [68] M. Fukugita and T. Yanagida, *Baryogenesis without grand unification*, *Phys. Lett. B* **174** (1986) 45.